DECTRIS®

detecting the future

π

π^e trillion digits of π

Peter Trüb, November 2016

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

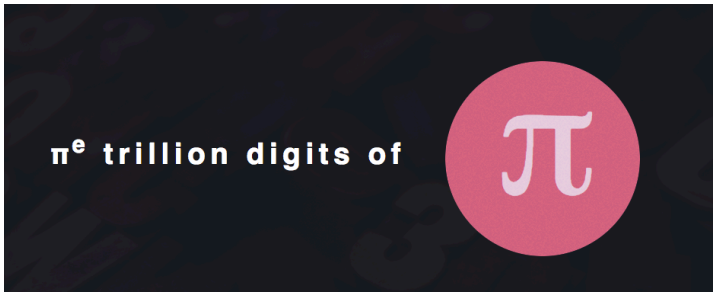
DECTRIS®

detecting the future

The Project

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

The Project



Goal Compute π with an new world record accuracy

Timeline

March 17	Two warm-up records with y-cruncher
April 7	Proposal
June 13	Approval by DECTRIS
July 5	Storage assembly
July 29	Computation start
Nov 11	Computation end
Nov 15	Announcement on numberworld.org

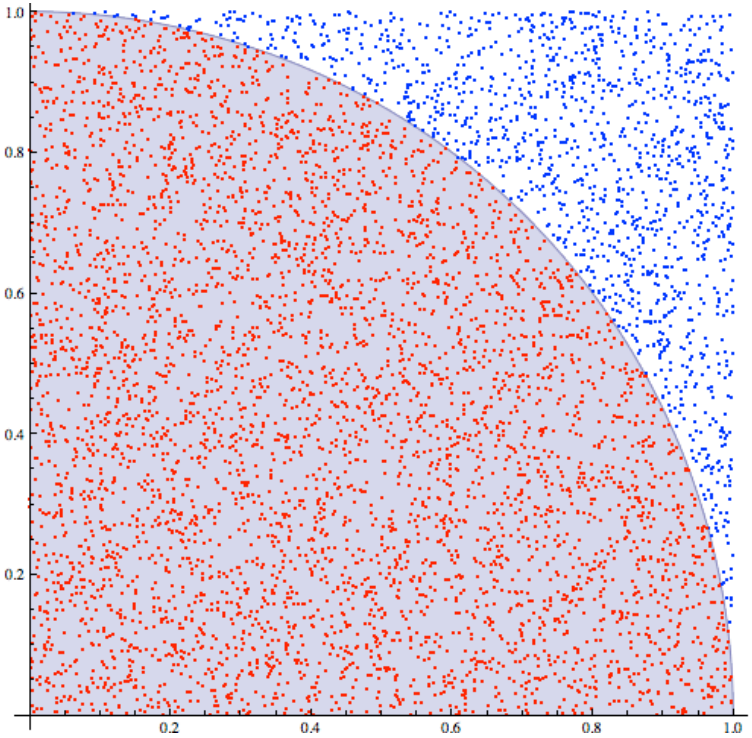
DECTRIS®

detecting the future

The Math

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

The Geometric Approach



$$\pi \approx 4 * \frac{number\ of\ red\ dots}{number\ of\ all\ dots}$$

Convergence: Slow

Try it https://repl.it/EYai/3

https://en.wikipedia.org/wiki/Pi#/media/File:Pi_30K.gif

The Leibniz Series

$$\frac{\pi}{4} = \arctan(1) = \int_0^1 \frac{1}{1+x^2}$$
$$= \int_0^1 1 - x^2 + x^4 - x^6 + \ldots$$
$$= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

Convergence: 10x more terms for each correct digit

Try it https://repl.it/C9Q2/8

The Chudnovsky Series

$$\frac{1}{\pi} = \frac{\sqrt{10005}}{4270934400} \sum_{k=0}^{\infty} (-1)^k \frac{(6k)!}{(k!)^3 (3k)!} \frac{(13591409 + 545140134k)}{640320^{3k}}$$

Convergence: 14 digits per term

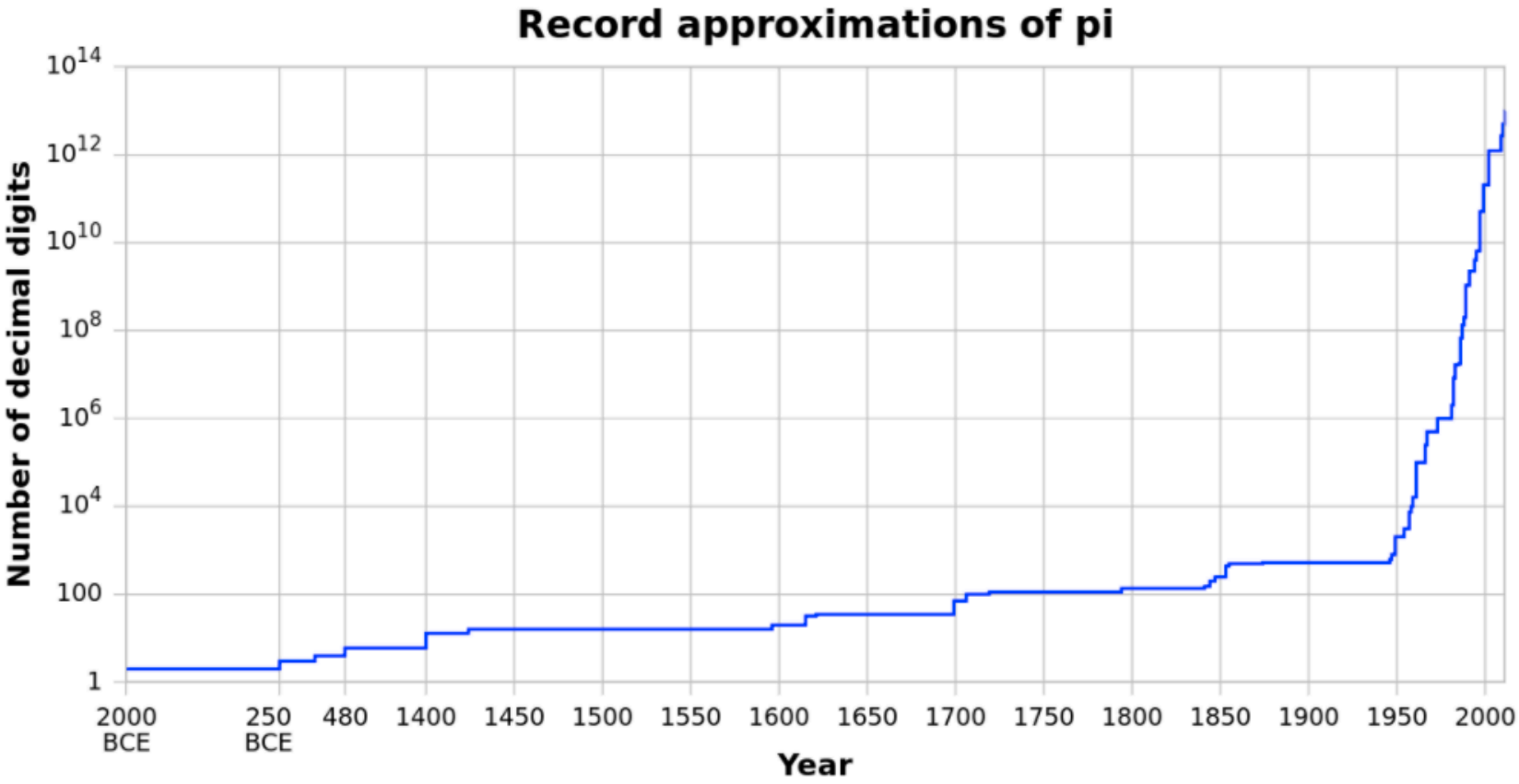
Try it https://repl.it/Cha7/12

Bellard's Formula

$$\pi = \frac{1}{2^6} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{10n}} \left(-\frac{2^5}{4n+1} - \frac{1}{4n+3} + \frac{2^8}{10n+1} - \frac{2^6}{10n+3} - \frac{2^2}{10n+5} - \frac{2^2}{10n+7} + \frac{1}{10n+9} \right)$$

Allows to compute the nth binary or hexadecimal digit without knowing the preceeding digits

History



https://en.wikipedia.org/wiki/Pi#/media/File:Record_pi_approximations.svg

DECTRIS®

detecting the future

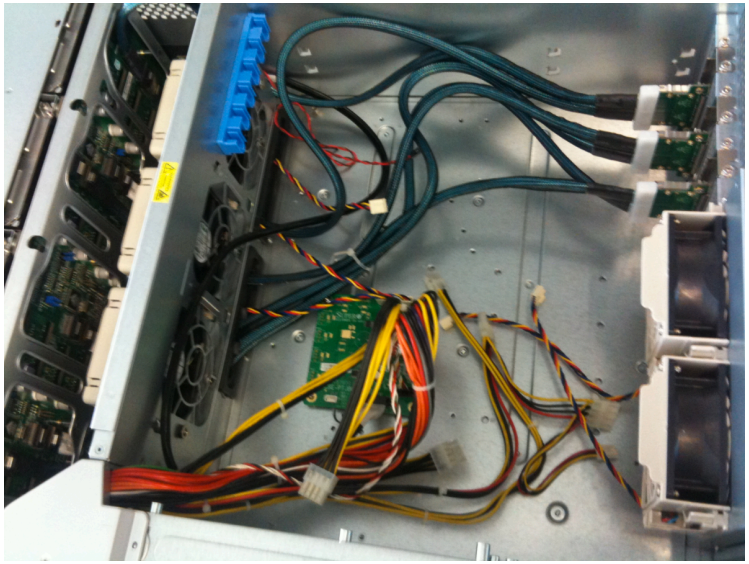
The Hardware

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

The Storage

24 x 6TB Seagate Enterprise NAS
- 7200 rpm
- SATA Interface
- 216 MB/s sustained transfer rate

3 x 8-Port SAS-Controllers



Storage Configuration

JBOD

- Just a bunch of disks (Non-RAID)
- Total transfer rate: 20 x 200MB/s ≈ 4GB/s
- No redundancy
- Regular backups

Device:	wkB/s a
sda	0.00
sdq	210944.00
sdo	211456.00
sdx	218112.00
sdp	204800.00
sdn	212480.00
sdu	203776.00
sdl	218624.00
sdy	211456.00
sdr	210432.00
sds	214528.00
sdw	220672.00
sdj	215040.00
sdv	217600.00
sdt	209408.00
sdf	216576.00
sdi	216064.00
sdb	203264.00
sdc	219648.00
sdh	205824.00
sdd	214528.00
sdg	204800.00
sde	220672.00
sdm	211456.00
sdk	216064.00

The Server

DELL R930
- 4 CPUs Intel Xeon 2.50 GHz
- 72 cores / 144 threads
- 1.25 TB RAM

DECTRIS®

detecting the future

The Software

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

Y-Cruncher

Closed Source 

Computes mathematical constants with several algorithms

π, e, ln(2), $\sqrt{2}$, Γ, $\zeta(3)$

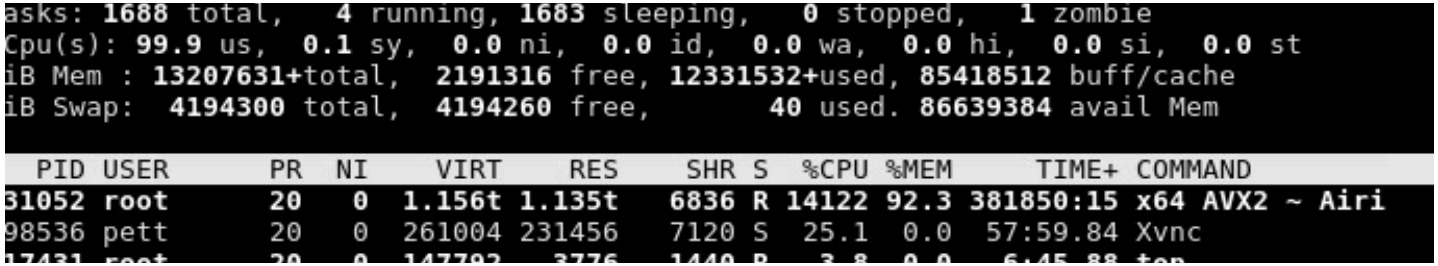
World records must be verified with second algorithm

Processor(s):	2 x Xeon X5482	2 x Xeon E5-2690[1]	2 x Xeon E5-2683 v3[1]	2 x Xeon E5-2696 v4[2]	4 x Xeon E7-8880 v3[3]
Generation:	Intel Penryn	Intel Sandy Bridge	Intel Haswell	Intel Broadwell	Intel Haswell
Cores/Threads:	8/8	16/32	28/56	44/88	64/128
Processor Speed:	3.2 GHz	3.5 GHz	2.03 GHz	2.2 GHz	2.3 GHz
Memory:	64 GB - 800 MHz	256 GB - ???	128 GB - ???	768 GB - ???	2 TB - ???
Version:	v0.6.9 - SSE4.1	v0.6.2/3 - AVX	v0.6.9 - AVX2	v0.7.1 - ADX	v0.7.1 - AVX2
25,000,000	6.578	2.283	0.907	0.715	1.176
50,000,000	12.965	4.295	1.745	1.344	2.321
100,000,000	26.194	8.167	3.317	2.673	4.217
250,000,000	71.395	20.765	8.339	6.853	8.781
500,000,000	157.234	42.394	17.708	14.538	15.879
1,000,000,000	347.962	89.920	37.311	31.260	32.078

Parallelization

Multithreading

Parallel computation on 4 CPU/72 Cores/144 Threads
Multi-core Efficiency: 22%
Threading on Windows is 5-10% faster than on Linux



Vectorization

Custom binaries for CPU vector extensions like SSE3, SSE4, AVX, AVX2

More on Parallelization

General

- Large computations are limited by disc throughput

GPU

- Optimized for floating point performance
- Large number arithmetic hard to vectorize (carry propagation)
- Bandwidth limitations

Clusters

- Communication bandwidth limitations

DECTRIS®

detecting the future

The Result

DECTRIS Ltd.
5400 Baden
Switzerland
www.dectris.com

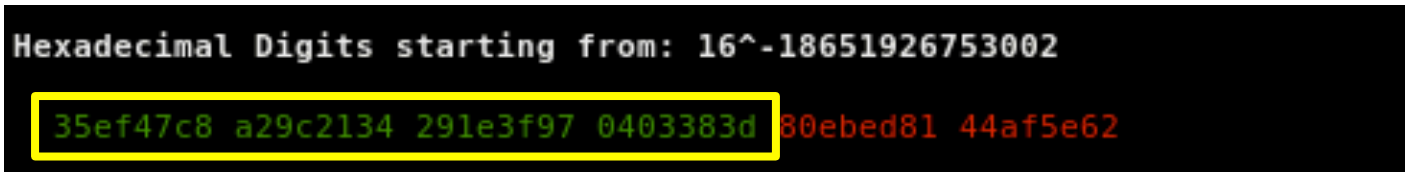
π

3.
1415926535 8979323846 2643383279 5028841971
6939937510 5820974944 5923078164 0628620899
8628034825 3421170679 8214808651 3282306647
0938446095 5058223172 5359408128 4811174502
8410270193 8521105559 6446229489 5493038196
4428810975 6659334461 2847564823 3786783165
2712019091 4564856692 3460348610 4543266482
1339360726 0249141273 7245870066 0631558817
4881520920 9628292540 9171536436 7892590360
0113305305 4882046652 1384146951 9415116094
…
4800693743 3394223303 9606068312 4988704460
0028555820 9734240923 7 …

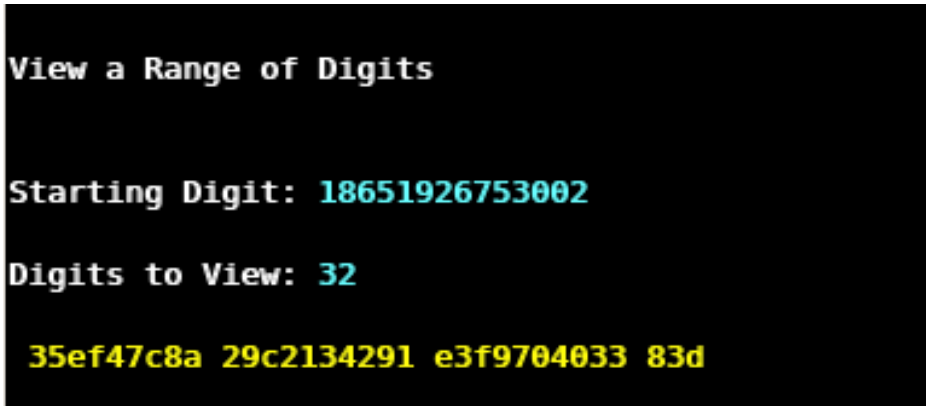
Verification

Hexadecimal digits verified with Bellard's formula

Bellard



Chudnovsky

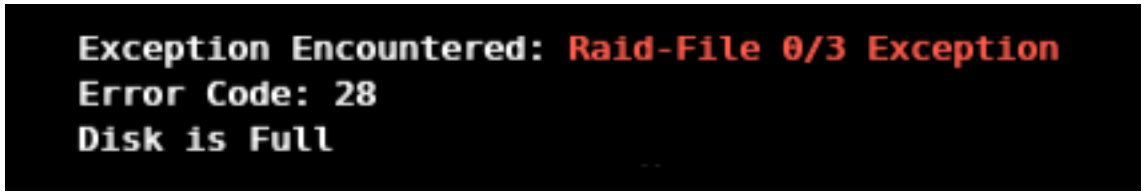


Decimal digits are verified by y-cruncher

Computation Challenges

0 disk failures
- Total bytes written ≈8 PB
- Data written per disk: ≈400 TB
- Data read ≈7 PB

Disk is full
- Confusion of TB (10^{12} bytes) with TiB (2^{40} bytes)



Normality of Pi

Every sequence occurs equally often in the (infinite) sequence of digits

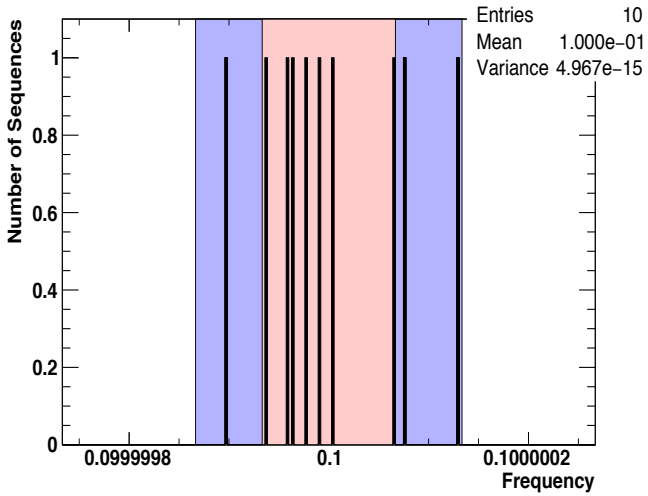
Examples

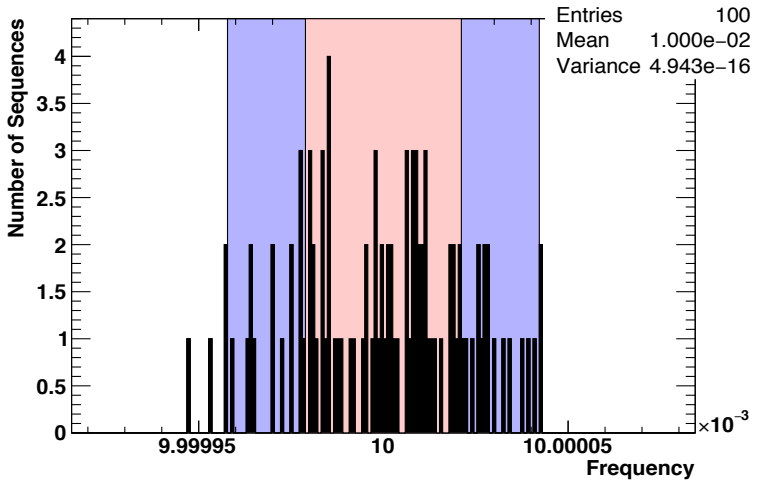
Every digit 0-9 occurs with frequency 1/10

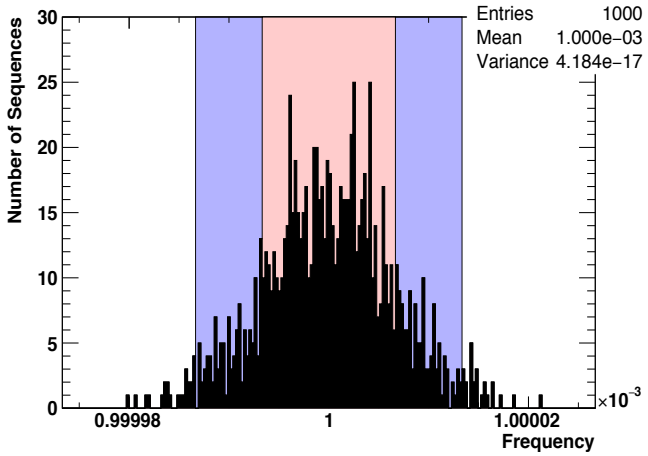
45 occurs with frequency of 1/100

3ac occurs with frequency of 1/4096

Normality of Pi



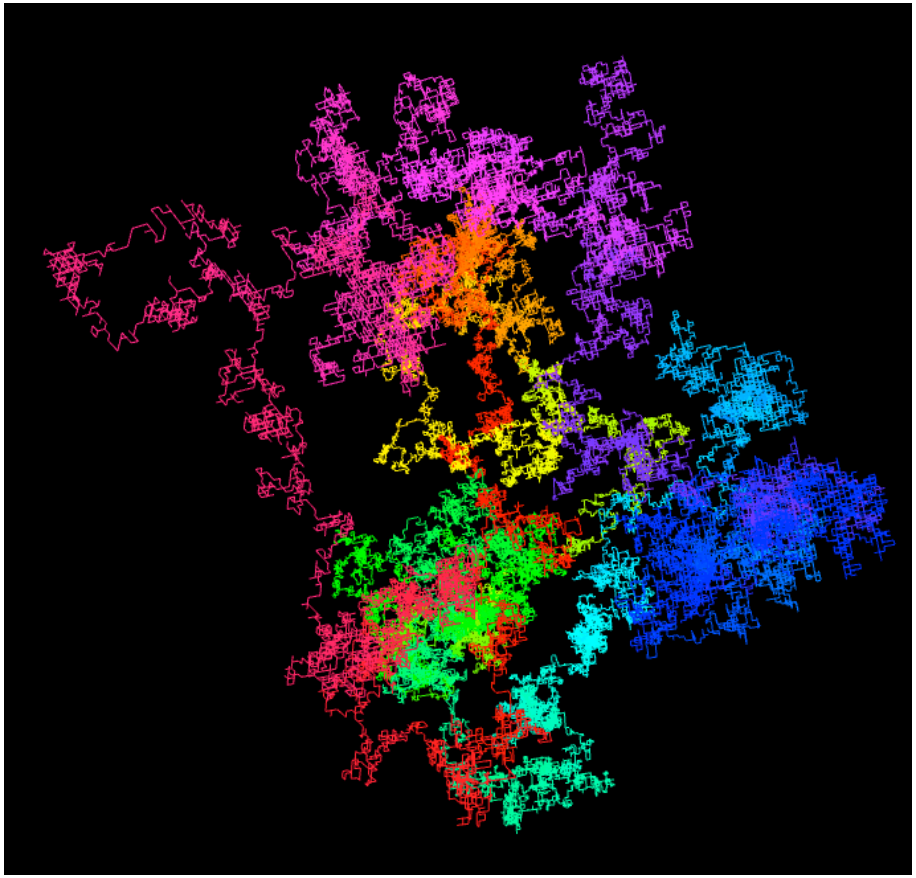




Pi looks very normal …

The Pi-Molecule

Pi can look very aesthetically …



Try it http://www.babylonjs-playground.com/#1TZEL3%236

Further Reading

Website www.dectris.com/successstories.html#success_pi

Blog www.pi2e.ch